

MOMENTUM TRANSFER DEPENDENCE OF GPDs

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Abstract. Based on the factorization representation of the General Parton Distributions (GPDs) the momentum transfer dependence was determined by the analysis of the different representations of parton distribution functions (PDFs) and all possible experimental data of the electromagnetic form factors of the proton and neutron. The obtained t -dependence of the GPDs is checked by analysis of the different hadronic reactions (including exclusive and elastic hadron scattering) in a wide energy region with minimum free fitting parameters.

1. Introduction

The parton picture of the hadron structure is represented, in most part, by the parton distribution functions (PDFs). They are determined in the deep inelastic processes. The next step in the development of the picture of the hadron structure was made by introducing the non-forward structure functions - general parton distributions - GPDs [1, 2, 3] with the spin-independent $H(x, \xi, t)$ and the spin-dependent $E(x, \xi, t)$ parts. Some of the advantages of GPDs were presented by the sum rules [2]

$$F_1^q(t) = \int_0^1 dx \mathcal{H}^q(x, \xi = 0, t), \quad F_2^q(t) = \int_0^1 dx \mathcal{E}^q(x, \xi = 0, t). \quad (1)$$

Using the different momenta of GPDs as a function of x^{n-1} we can obtain the different form factors.

$$F_{n-1}^q(t) = \int_0^1 dx x^{n-1} \mathcal{F}^q(x, \xi = 0, t) \quad (2)$$

$n = 0$, (zero moment) give the Compton form factors $R_V(t), R_A(t), R_T(t)$
 $n = 1$, (first moment) give the electromagnetic form factors $F_1(t), F_2(t)$
 $n = 2$, (second moment) give the gravimagnetic form factors $A_1(t), B_2(t)$

Good knowledge of the momentum transfer dependence of the GPDs is required. Now we cannot obtain the t -dependence of GPDs from the first principles, but it can be obtained from the phenomenological description by *GPDs* of the nucleon electromagnetic form factors. Many different forms of the t -dependence of GPDs were proposed. In the quark diquark model [4, 5]

the form of GPDs consists of three parts - PDFs, function distribution and Regge-like. In other works (see e.g. [6]), the description of the t -dependence of GPDs was developed in a more complicated picture using the polynomial forms with respect to x . In [7], it was shown that at large $x \rightarrow 1$ and momentum transfer the behavior of GPDs requires a larger power of $(1-x)^n$ in the t -dependent exponent:

$$\mathcal{H}^q(x, t) \sim \exp[a (1-x)^n t] q(x). \quad (3)$$

with $n \geq 2$. It was noted that $n = 2$ naturally leads to the Drell-Yan-West duality between parton distributions at large x and form factors.

2. New momentum transfer dependence of GPDs

Let us modify the original Gaussian ansatz and choose the t -dependence of GPDs in a simple form

$$\mathcal{H}^q(x, t) = q(x) \exp[a_+ (1-x)^2/x^m t]. \quad (4)$$

The value of the parameter $m = 0.4$ is fixed by the low t experimental data while the free parameters a_{\pm} (a_+ - for \mathcal{H} and a_- - for \mathcal{E}) were chosen to reproduce the experimental data in the whole t region. The isotopic invariance can be used to relate the proton and neutron GPDs. Hence, we do not change any parameter and keep the same t -dependence of GPDs as in the case of proton.

In all calculations we restrict ourselves to the contributions of only valence u and d quarks. In our first work [8] the function $q(x)$ is based on the MRST02 global fit [9].

Further development of the model requires a careful analysis of the momentum transfer form of GPDs and properly chosen PDFs form. In [10], the analysis of more than 24 different PDFs was made. The complex analysis of the corresponding description of the electromagnetic form factors of the proton and neutron by the different PDF sets (24 cases) was carried out. These PDFs include the leading order (LO), next leading order (NLO) and next-next leading order (NNLO) determination of the parton distribution functions. They used the different forms of the x dependence of PDFs. We slightly complicated the form of GPDs in comparison with eq.(4), but it is the simplest one as compared to other works (for example [11]).

$$\mathcal{H}^u(x, t) = q(x)_{nf}^u e^{2a_H \frac{(1-x)^{2+\epsilon_u}}{(x_0+x)^m} t}; \quad \mathcal{H}_{\lceil}^d(x, t) = q(x)_{nf}^d e^{2a_H(1+\epsilon_0)(\frac{(1-x)^{1+\epsilon_d}}{(x_0+x)^m}) t}. \quad (5)$$

$$\mathcal{E}^u(x, t) = q(x)_{fl}^u e^{2a_E \frac{(1-x)^{2+\epsilon_u}}{(x_0+x)^m} t}; \quad \mathcal{E}_{\lceil}^d(x, t) = q(x)_{fl}^d e^{2a_E(1+\epsilon_0)(\frac{(1-x)^{1+\epsilon_d}}{(x_0+x)^m}) t}. \quad (6)$$

where $q(x)_{fl}^{u,d} = q(x)_{nf}^{u,d} (1-x)^{z_1, z_2}$

To obtain the form factors, we have to integrate over x in the whole range $0-1$. Hence, the form of the x -dependence of PDF affects the form and size of the form factor. But the PDF sets are determined from the inelastic processes only in some region of x , which is only approximated to $x = 0$ and $x = 1$. Some PDFs have the polynomial form of x with different power. Some other have the exponential dependence of x . As a result, the behavior of PDFs, when $x \rightarrow 0$ or $x \rightarrow 1$, can impact the form of the calculated form factors. The analysis was carried out with different forms of the t dependence of GPDs. The minimum number of free parameters was six and maximum was ten. The obtained electromagnetic form factors and the ratio of the electromagnetic form factors for the proton $\mu_p G_E^p/G_M^p$ and for the neutron $\mu_n G_E^n/G_M^n$ were described quantitatively well. Our calculation reproduces the data obtained by the polarization method.

On the basis of our GPDs with ABM12 [12] PDFs we calculated the hadron form factors by the numerical integration

$$F_1(t) = \int_0^1 dx \left[\frac{2}{3} q_u(x) e^{2\alpha_H t(1-x)^{2+\epsilon_u}/(x_0+x)^m} - \frac{1}{3} q_d(x) e^{2\alpha_H t(1-x)^{1+\epsilon_d}/((x_0+x)^m)} \right] \quad (7)$$

and then by fitting these integral results by the standard dipole form with some additional parameters for $F_1(t)$

$$F_1(t) = \frac{4m_p - \mu t}{4m_p - t} \frac{1}{(1 + q/a_1 + q^2/a_2^2 + q^3/a_3^3)^2} \quad (8)$$

The matter form factor

$$A(t) = \int_0^1 x dx [q_u(x) e^{2\alpha_H t(1-x)^{2+\epsilon_u}/(x_0+x)^m} + q_d(x) e^{2\alpha_H t(1-x)^{1+\epsilon_d}/((x_0+x)^m)}] \quad (9)$$

is fitted by the simple dipole form $A(t) = \Lambda^4/(\Lambda^2 - t)^2$.

Our description is valid up to a large momentum transfer with the following parameters: $a_1 = 16.7$ GeV, $a_2^2 = 0.78$ GeV², $a_3^3 = 12.5$ GeV³ and $\Lambda^2 = 1.6$ GeV². These form factors will be used in our model of the proton-proton and proton-antiproton elastic scattering.

3. The Compton cross sections

Our calculations are based on the works [13, 11]. The differential cross section for that reaction can be written as

$$\frac{d\sigma}{dt} = \frac{\pi\alpha_{em}^2 (s-u)^2}{s^2 - us} [R_V^2(t) - \frac{t}{4m^2} R_T^2(t) + \frac{t^2}{(s-u)^2} R_A^2(t)], \quad (10)$$

where $R_V(t)$, $R_T(t)$, $R_A(t)$ are the form factors given by the $1/x$ moments of the corresponding GPDs $H^q(x, t)$, $E^q(x, t)$, $\tilde{H}^q(x, t)$. The last is related with the axial form factors. As noted in [11], this factorization, which bears some similarity to the handbag factorization of DVCS, is formulated in a symmetric frame where the skewness $\xi = 0$. For $H^q(x, t)$, $E^q(x, t)$ we used the PDFs obtained from the works [14] with the parameters obtained in our fitting procedure of the description of the proton and neutron electromagnetic form factors in [10].

$$R_V(t) = \int_0^1 \frac{dx}{x} \left\{ \frac{4}{9} u(x) \exp[2\alpha_1 \frac{(1-x)^{p_1}}{(x_0+x)^{p_1^2}} t] + \frac{1}{9} d(x) \exp[2\alpha_1 \frac{(1-x)^{p_1(k_d)}}{(x_0+x)^{p_2}} t] \right\}, \quad (11)$$

$$R_T(t) = \int_0^1 \frac{dx}{x} \left\{ \frac{4}{9} u^e(x) \exp[2\alpha_1 \frac{(1-x)^{p_1}}{(x_0+x)^{p_1^2}} t] + \frac{1}{9} d^e(x) \exp[2\alpha_1 \frac{(1-x)^{p_1(k_d)}}{(x_0+x)^{p_2}} t] \right\}, \quad (12)$$

$$R_A(t) = \int_0^1 \frac{dx}{x} \left\{ \frac{4}{9} \Delta u^e(x) \exp[2\alpha_1 \frac{(1-x)^{p_1}}{(x_0+x)^{p_1^2}} t] + \frac{1}{9} \Delta d^e(x) \exp[2\alpha_1 \frac{(1-x)^{p_1(k_d)}}{(x_0+x)^{p_2}} t] \right\}, \quad (13)$$

For $\tilde{H}^q(x, t)$ we take Δq^e in the form

$$\Delta q^e = N_i x_1^a (1 + a_2 \sqrt{x} + a_3 x), \quad (14)$$

with the parameters determined in [15]. The calculations of R_i on the whole, correspond to the calculations [11], but the integrals with our t dependence of GPDs do not diverge at momentum transfer $-t > 2$ GeV². In [11], R_i is presented beginning with $-t = 4$ GeV².

The calculations of the differential cross sections of RCS are shown in Fig.15 at three energies $s = 9.8, 10.92$ and 20 GeV². The calculations have a sufficiently good coincidence with the existing experimental data and, in some part, coincide with calculations [11].

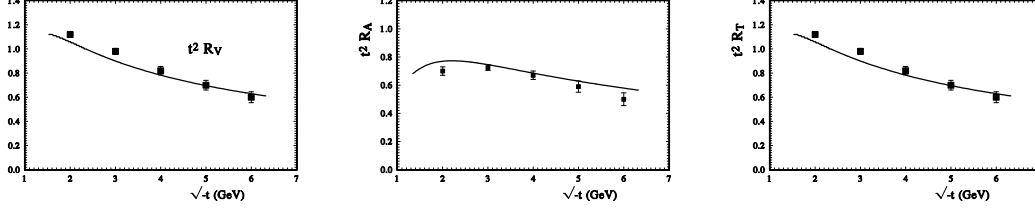


Figure 1. Compton form factors (squares - show the model calculations [11])
a) [left] $t^2 R_V(t)$, b) [middle] $t^2 R_T(t)$, c) [right] $t^2 R_A(t)$

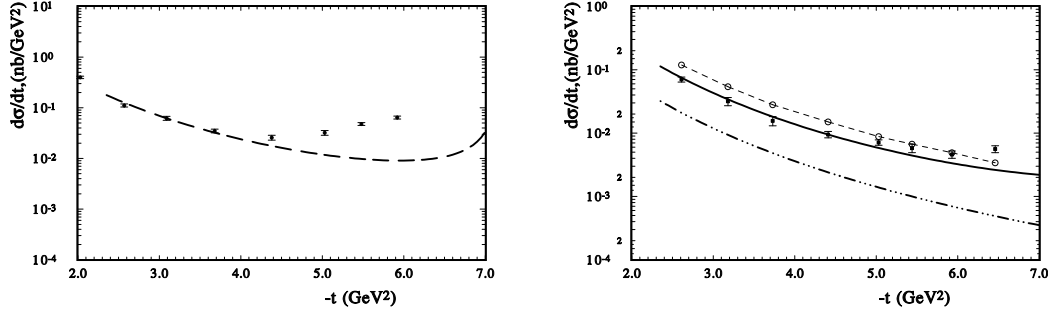


Figure 2. Differential Compton cross sections $\gamma p \rightarrow \gamma p$; the curves are our calculations at
a) [left] $s = 8.9 \text{ GeV}^2$, b) [right] $s = 10.92 \text{ GeV}^2$ (dashed line with circles - the calculations [11]), and $s = 20 \text{ GeV}^2$ the data points are for $s = 8.9 \text{ GeV}^2$ (circles); $s = 10.92 \text{ GeV}^2$ (squares) [23].

4. Elastic nucleon scattering (HEGS model)

Let us use the obtained momentum transfer dependence of GPDs in calculations of the electromagnetic and gravimagnetic form factors of the nucleons as the first and second moments of GPDs. The obtained form factors are related to the charge and matter distributions. The new High Energy General Structure (HEGS) model of the elastic proton-proton and proton-antiproton scattering was proposed [16, 17] with the two form factors determined early with the fixed parameters.

The model is very simple from the viewpoint of the number of parameters and functions. There are no any artificial functions or any cuts which bound the separate parts of the amplitude by some region of momentum transfer or energy. One of the most remarkable properties is that the real part of the hadron scattering amplitude is determined only by complex energy \hat{s} that satisfies the crossing-symmetries.

The differential cross sections of nucleon-nucleon elastic scattering can be written as the sum of different helicity amplitudes:

$$\frac{d\sigma}{dt} = \frac{2\pi}{s^2} (|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2). \quad (15)$$

The total helicity amplitudes can be written as $\Phi_i(s, t) = F_i^h(s, t) + F_i^{\text{em}}(s, t)e^{\varphi(s, t)}$, where $F_i^h(s, t)$ comes from the strong interactions, $F_i^{\text{em}}(s, t)$ from the electromagnetic interactions

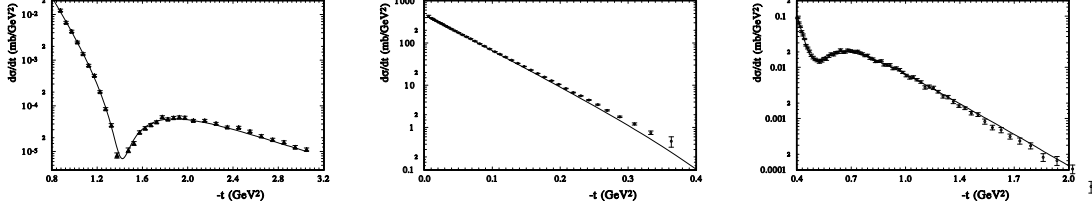


Figure 3. Differential cross sections of the elastic pp scattering a) [left] at $\sqrt{s} = 30.5$ GeV b)[middle] at $\sqrt{s} = 7$ TeV (small t) and c) [right] at $\sqrt{s} = 7$ TeV (large t)

and $\varphi(s, t)$ is the interference phase factor between the electromagnetic and strong interactions [18, 19, 20].

The (HEGS) model [17] gives a quantitative description of the elastic nucleon scattering at high energy with only 5 fitting high energy parameters. A successful description of the existing experimental data by the model shows that the elastic scattering is determined by the generalized structure of the hadron. The model leads to a coincidence of the model calculations with the preliminary data at 8 TeV. We found that the standard eikonal approximation [21] works perfectly well from $\sqrt{s} = 9$ GeV up to 8 TeV.

In the model the Born term of the elastic hadron amplitude is determined

$$F_h^{Born}(s, t) = h_1 F_1^2(t) F_a(s, t) (1 + r_1/\hat{s}^{0.5}) + h_2 A^2(t) F_b(s, t) (1 + r_2/\hat{s}^{0.5}), \quad (16)$$

where $F_a(s, t)$ and $F_b(s, t)$ have the standard Regge form

$$F_a(s, t) = \hat{s}^{\epsilon_1} e^{B(s)t}; \quad F_b(s, t) = \hat{s}^{\epsilon_1} e^{B(s)/4 t}. \quad (17)$$

The final elastic hadron scattering amplitude is obtained after unitarization of the Born term. So, first, we have to calculate the eikonal phase and then, using the standard eikonal representation, obtained the final hadron elastic scattering amplitude.

Our complete fit of 3416 experimental data in the energy range $9.8 \leq \sqrt{s} \leq 8000$ GeV and the region of the momentum transfer $0.000375 \leq -t \leq 14.75$ GeV² gave $\sum_{i=1}^N \chi_i^2/N = 1.28$ with the parameters $h_1 = 3.67$; $h_2 = 1.39$; $h_{odd} = 0.76$; $k_0 = 0.16$; $r_0^2 = 3.82$; and the low energy parameters $h_{sf} = 0.05$; $r_1 = 53.7$; $r_2 = 4.45$.

Obviously, for such a huge energy region we have a very small number of free parameters. Noteworthy also is a good description of the CNI region of momentum transfer in a very wide energy region, approximately three orders, with the same slope of the scattering amplitude. As example, the differential cross sections of the proton-proton elastic scattering in the region of the diffraction minimum and different energies are presented in Fig. 3 at $\sqrt{s} = 30.5$ GeV and $\sqrt{s} = 7$ TeV. In most part, the form and energy dependence of the diffraction region is determined by the real part of the scattering amplitude. Note that in the model only the Born term is determined. The complicated diffractive structure is obtained only after the unitarization procedure. The extended variant of the model [10] shows the contribution of the "maximal" odderon with specific kinematic properties and does not show a visible contribution of the hard pomeron, as in [22].

Only our model takes into account the whole region of the momentum transfer ($3.75 \cdot 10^{-4} \geq |t| \geq 15$ GeV²), which includes the high precision experimental data in the Coulomb-hadron interference region.

5. Conclusion

The analysis of PDFs of many Collaborations with different forms of the x -dependence show that additional parameters of the function $f(x)$ in the exponential t -dependence part of GPDs lead to the identical results in the final fitting procedure. Hence, such additional parameters equalize the different form of PDFs to obtain the same results for the electromagnetic form factors of the nucleons.

Our examination of the obtained momentum transfer dependence of GPDs was checked in the case of the real Compton scattering. The calculated Compton form factors $R_V(t)$, $R_T(t)$, $R_A(t)$ gave a somewhat larger slope than such form factors calculated in [11] (see Fig.1). Our form factors reproduce the differential cross sections of the real Compton scattering at large momentum transfer reasonably well (see Fig.2).

GPDs let us calculate not only electromagnetic form factor (which reflects the charge distribution) but also the second moment of the GPDs give the gravimagnetic form factors (which reflects the matter distribution of the nucleons). The new HEGS model of the elastic pp and $\bar{p}p$ scattering was built taking into account both these form factors. It gave the possibility to essentially decrease the number of the fitting parameters. As a result, the model with a minimum of the fitting parameters describes quantitatively the maximum number of the experimental data in a wide region of the energy ($9.8 \text{ GeV} \leq \sqrt{s} \leq 8 \text{ TeV}$) and the momentum transfer ($0.00037 \leq |t| \leq 15 \text{ GeV}^2$). Such a unique description of the differential cross sections on the basis of the GPDs gave large support of the determining momentum transfer dependence of the GPDs.

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